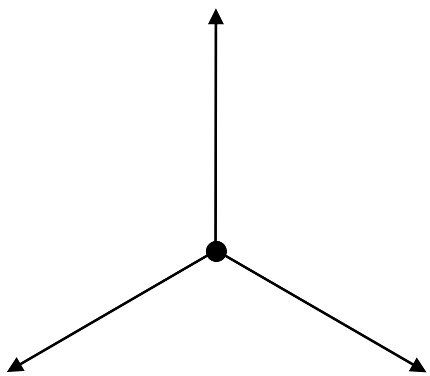
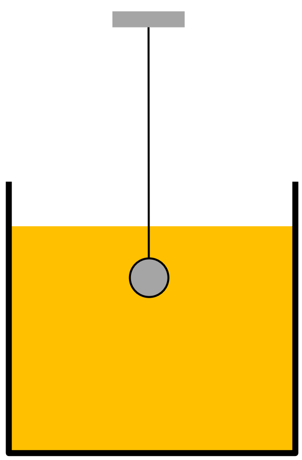
Problem of the week
Forces

(a) Three coplanar forces of equal magnitude 25 N act on a body. The angle between any two adjacent forces is 120°.



Determine the resultant force on the body.

(b) A small spherical body of density 2200 kg m^{-3} and radius 0.50 cm is attached to a vertical string and is fully immersed in a liquid of density 1100 kg m^{-3}. The body is in equilibrium.



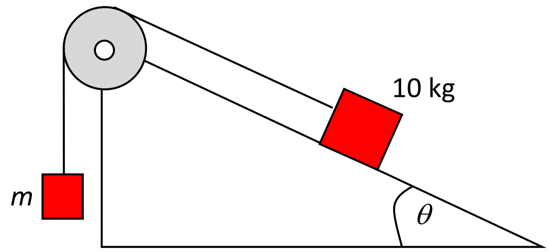
(i) Draw and label the forces on the body.
(ii) Calculate the tension in the string.

(c) The string is cut. Calculate the initial acceleration of the body.

(d)

(i) The container is very deep. The liquid is viscous with a viscosity of 56 mPa s. Explain why the body will reach terminal speed.
(ii) Estimate the terminal speed.

(e) A block of mass M = 10.0 kg rests on a rough inclined plane attached to a hanging block of mass m through a pulley as shown. The incline makes an angle $\theta = 30^{\circ}$ with the horizontal. The static coefficient of friction between the block and the incline is 0.40 and the kinetic coefficient is 0.30.



(i) Determine the largest and the smallest mass m that can hang from the string so that we have equilibrium.

(ii) The hanging mass is 5.0 kg. Calculate the frictional force acting on the 10 kg block.

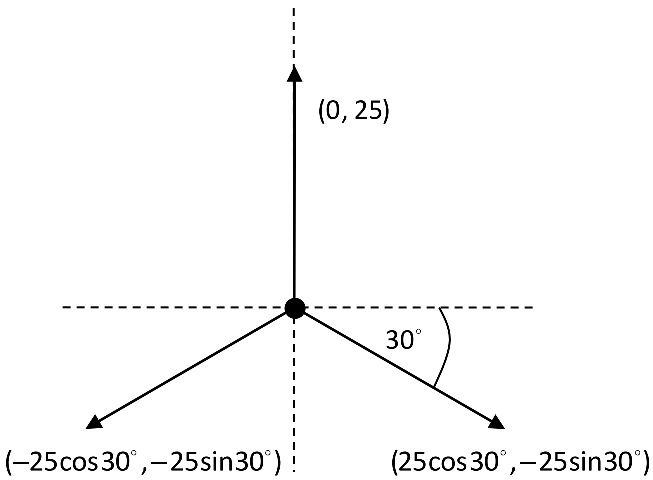
(f) The string in (e) is cut.

(i) Calculate the acceleration of the 10 kg block.

(ii) Estimate the time it will take the block to move 4.0 m down the inclined plane.

Answers

(a) The components along the axes shown are:



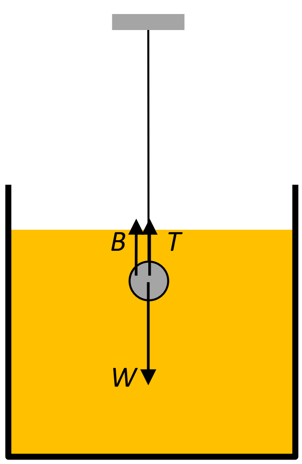
The resultant then has components:

$$(0+25\cos 30^\circ - 25\cos 30^\circ,\ 25-25\sin 30^\circ - 25\sin 30^\circ) = (0,\ 25-\frac{25}{2}-\frac{25}{2}) = (0,0)\ .$$

The resultant is zero.

(b)

(i) *W*eight down, *T*ension up, *B*uoyant up.



(ii)

$$T + B = mg$$
$$T = mg - B$$
$$T = \frac{4\pi r^3}{3}\rho_{\text{body}}g - \frac{4\pi r^3}{3}\rho_{\text{liquid}}g$$
$$T = \frac{4\pi r^3}{3}g(\rho_{\text{body}} - \rho_{\text{liquid}})$$
$$T = \frac{4\pi(0.50\times 10^{-2})^3}{3}\times 9.8\times(2200-1100)$$
$$T = 5.64 \approx 5.6 \text{ mN}$$

(c) The resultant force is $mg - B$ and so 5.64 mN. The mass is

$m = \frac{4\pi r^3}{3}\rho_{\text{body}} = \frac{4\pi(0.50\times 10^{-2})^3}{3}\times 2200 = 1.152\times 10^{-3}$ kg so the acceleration is

$$a = \frac{5.64\times 10^{-3}}{1.152\times 10^{-3}} = 4.9 \text{ m s}^{-2}$$

OR

$$ma = mg - B \Rightarrow a = g - \frac{B}{m} = g - \frac{\rho_{\text{liquid}}Vg}{\rho_{\text{body}}V} = g(1 - \frac{\rho_{\text{liquid}}}{\rho_{\text{body}}}) = 4.9 \text{ m s}^{-2}$$

(d)

(i) A drag force *D* proportional to speed (Stokes) will act on the body opposing the motion. Eventually the drag force will increase sufficiently so that the net force on the body $(mg - B - D)$ will be zero.

(ii) $mg - B - D = 0$. Hence

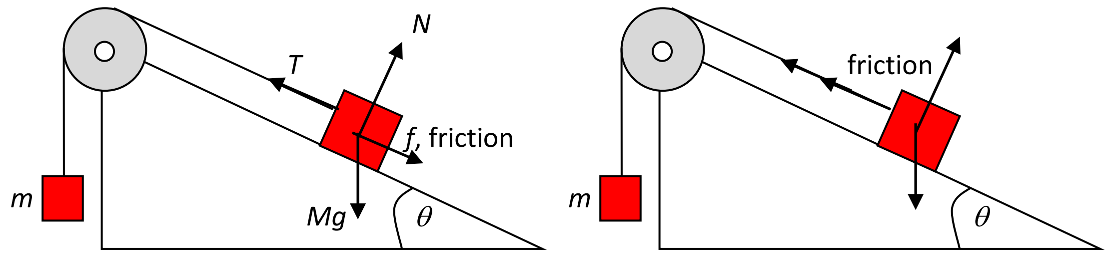
$$6\pi\eta r v = \frac{4\pi r^3}{3}\rho_{\text{body}}g - \frac{4\pi r^3}{3}\rho_{\text{liquid}}g$$
$$v = \frac{2r^2}{9\eta}g(\rho_{\text{body}} - \rho_{\text{liquid}})$$
$$v = \frac{2\times(0.50\times 10^{-2})^2}{9\times 0.056}\times 9.8\times(2200-1100)$$
$$v = 1.1 \text{ m s}^{-1}$$

OR

$$6\pi\eta r v = 5.64\times 10^{-3}$$
$$v = \frac{5.64\times 10^{-3}}{6\pi\times 0.056\times 0.50\times 10^{-2}}$$
$$v = 1.1 \text{ m s}^{-1}$$

(e)

(i) The forces when the largest and smallest possible *m* are acting are:



Largest   Smallest

In both cases the tension in the string is *mg*.

Largest *m*: $T = Mg\sin\theta + f_{max} = Mg\sin\theta + \mu N = Mg\sin\theta + \mu Mg\cos\theta$. Hence $mg = Mg\sin\theta + \mu Mg\cos\theta$ and finally $m = 10\times\sin 30^\circ + 0.40\times 10\times\cos 30^\circ = 8.5\text{ kg}$.

Smallest *m*: $T = Mg\sin\theta - f_{max} = Mg\sin\theta - \mu N = Mg\sin\theta - \mu Mg\cos\theta$. Hence $mg = Mg\sin\theta - \mu Mg\cos\theta$ and finally $m = 10\times\sin 30^\circ - 0.40\times 10\times\cos 30^\circ = 1.5\text{ kg}$.

(ii) The tension in the string is $mg = 5.0\times 9.8 = 49\text{ N}$. The component of the weight down the plane is $Mg\sin 30^\circ = 49\text{ N}$. Hence the frictional force is zero.

(f)

(i) The resultant force is $Mg\sin\theta - f = Mg\sin\theta - \mu_k N = Mg\sin\theta - \mu_k Mg\cos\theta$ and so the acceleration is $a = g(\sin\theta - \mu_k\cos\theta) = 2.354 \approx 2.4\text{ m s}^{-2}$.

(ii) From $s = \frac{1}{2}at^2$, $t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2\times 4.0}{2.354}} = 1.8\text{ s}$.